Efficient Simulation of Ricean Fading within a Packet Simulator

Ratish J. Punnoose, Pavel V. Nikitin, and Daniel D. Stancil
Department of Electrical and Computer Engineering
Carnegie Mellon University
Pittsburgh PA 15213
{punnoose,pnikitin,stancil}@andrew.cmu.edu

Abstract

Packet level network protocol simulators use simple channel models for computational efficiency. A typical method for doing this is to compute a packet error probability assuming a certain fading distribution without taking into account time-correlation. This paper introduces work that has been done to model the effect of small-scale fading (Rayleigh and Ricean) within the ns network simulator. It allows for the faithful simulation of a complete fading envelope. The fading models have the appropriate statistics and also time correlational properties obtained from the Doppler spectrum. An efficient implementation based on a simple table lookup is described.

1. Introduction

Hardware based fading simulators using analog or digital components have been described extensively in the literature [1, 9, 4, 2]. These are particularly convenient for testing prototype devices and reproducing environment conditions in the lab. But software based fading simulators are more convenient in testing a design or a mathematical model.

Wireless network protocol simulators that operate at a packet level are computationally intensive. The addition of accurate channel fading models must add as little complexity as possible. In a typical simulation, the effects of fading are approximated using random number generation with the appropriate statistical behavior. The drawback of such an approach is that it does not describe the time-correlation of the signal envelope. It is important to model the effects of time-correlation to accurately simulate burst errors. Time-correlation can be achieved by filtering generated random numbers with the Doppler spectrum. However, this increases the computational requirements and adds an additional level of complexity since packet arrivals may not be at uniform time intervals.

This paper introduces a simple method of modeling small scale Ricean (or Rayleigh) fading. A dataset containing the components of a time-sequenced fading envelope is pre-computed. With a few simple mathematical operations during the simulation run, this single lookup table can be used to model a wide range of parameters. The parameters to be adjusted are the time-averaged power, P, the maximum Doppler frequency, f_m, and the Ricean K factor. Although the dataset represents a limited length time-sequence, long simulations can be performed by using this limited dataset over and over again. The dataset is constructed so that there are no discontinuities when the sequence repeats. This approach is simpler than building a fading simulator using dedicated digital or analog hardware.

It is assumed that the small scale fading envelope is used to modulate the calculations of a large scale propagation model (two-ray ground or some other deterministic model).

Section 2 describes the creation of the dataset and its use with a large scale model. Section 3 details the use of a single dataset to simulate a range of parameters. Section 4 gives an example of the simulator output and analyzes the simulated fading envelopes for statistical accuracy.

2. Pre-computing a dataset for the fading envelope

Small scale fading is caused by movement of the transmitter, receiver, or of other objects in the environment. This motion can be characterized by the Doppler spreading. Fading can be modeled by using frequency-domain random numbers with the appropriate statistics and then applying time-correlation by performing spectral shaping on these data using the Doppler spread.

We will concentrate on the the Ricean distribution and treat the Rayleigh distribution as a special case (K=0).

Computation of the dataset is performed as described in many places in the literature[8], [7]. In-phase and quadrature phase components are generated using a Gaussian distribution, then spectrally shaped by the Doppler spectrum.

The model of Clarke and Gans is used for the Doppler power spectral density [3], [5]. The dataset essentially contains a sequence of in-phase and quadrature components which are then combined (with the appropriate Ricean K-factor) within the simulator to yield the fading envelope.

The following steps are taken to create the dataset.

1. Specify the number of points, the maximum Doppler frequency, f_m, and the frequency spacing of the points used. The frequency spacing determines the time duration.

2. Generate complex Gaussian random numbers for each of the frequency components. The complex frequency component at any particular frequency and at its negative frequency should be complex conjugates. This will yield a real waveform.

3. Create the fading spectrum, evaluated at the same frequency points as above. This can be as obtained from the Gans model.

4. Multiply the frequency domain components with the fading spectrum.

5. Perform an IFFT on this data to yield a time series data.

6. Repeat steps 2,3,4, and 5 to give a second time series. These two sets form the in-phase and quadrature phase multipath sources.

The process of performing the IFFT on this low-pass filtered frequency domain data ensures that the beginning and end of the dataset if wrapped around, do not have discontinuities.

For the purposes of simulation, it is convenient to store the in-phase and quadrature-phase multipath sources, x_1, x_2, normalized to have unity variance.

A Ricean amplitude with parameter K can be created by combining the quadrature components x_1, x_2, from the dataset as

$$r = \sqrt{(\sigma x_1 + A)^2 + \sigma x_2^2} \tag{1}$$

where $A^2 / (2\sigma^2) \equiv K$, and σx_i is a random variable with variance σ^2.

The mean-squared value of the Ricean distribution is given by

$$A^2 + 2\sigma^2 = 2\sigma^2 (K + 1) \tag{2}$$

The normalized power envelope is therefore

$$\frac{r^2}{P} = \frac{1}{2(K+1)} \left[\left(x_1 + \sqrt{2K} \right)^2 + x_2^2 \right]. \tag{3}$$

This power envelope can be used to modulate the output of a large scale propagation model. The above description assumes that the mean squared value of the envelope is

the power predicted by the large scale model. Specifically, in the Ricean model, this power contains the power in the dominant path and the multipath power. With certain propagation models, it may be more appropriate for the large scale power calculation to represent only the value in the dominant path. In this case, Eqn 1 must be divided by the factor $A^2 = 2\sigma^2 K$. The normalized power envelope then takes the form

$$\frac{r^2}{P} = \frac{1}{2K} \left[\left(x_1 + \sqrt{2K} \right)^2 + x_2^2 \right], \tag{4}$$

where P is the large scale model expression for the power in the dominant path.

3. Time-stretching for simulation with different Doppler frequencies

In the previous section, it was shown how the amplitude of the envelope could be adjusted to match the power predicted by a large scale propagation model. The dataset that has been created can also be time scaled to match different values of the maximum Doppler frequency.

The Doppler filter from the Gans model is given as

$$S(f) = \frac{1.5}{\pi f_m \sqrt{1 - \left(\frac{f - f_c}{f_m} \right)^2}} \tag{5}$$

or

$$S(F) = \frac{1.5}{\pi f_m \sqrt{1 - \left(\frac{F}{f_m} \right)^2}} \tag{6}$$

where $F = f - f_c$, is frequency relative to the carrier.

The value of f_m outside the square root sign changes the gain of the Doppler filter but since the filtered data, x_1, x_2, are normalized, this need not be taken into account while time-scaling.

Let

$$\frac{f_{m_0}}{F_0} = k \tag{7}$$

where k is a constant fixed while creating the data set, f_{m_0} is the maximum Doppler frequency and F_0 is the largest frequency component represented while creating the data set (the largest value of F in Eqn 6). Frequency F_0 is determined by the number of points and the frequency spacing of the points. F_0 has to be greater than f_{m_0}. Changing the ratio k changes the resolution of the created envelope. The value of k only has to be chosen once in such a manner that the resolution is adequate to smoothly represent the envelope for that set of parameters.

Let the time-spacing of the dataset be Δt_0 defined as

$$\Delta t_0 = \frac{1}{F_0}. \tag{8}$$
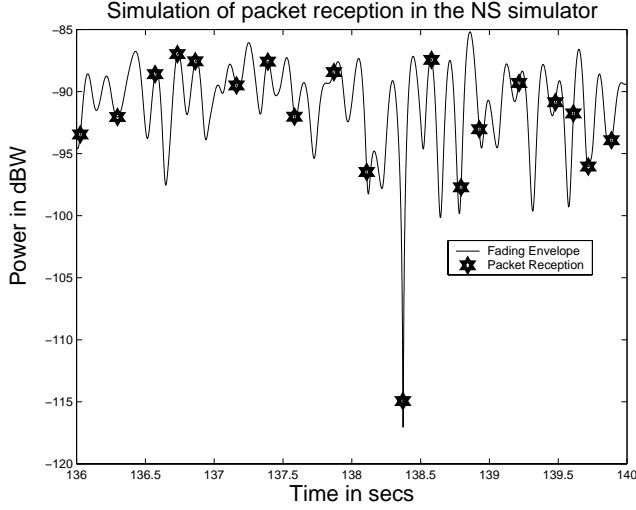
Figure 1. Simulated Received Packet Signal Strength, (K=7.8dB, f_m=7.625 Hz, transmitter power = 0.281 W, Approximate distance between transmitter and receiver at time 138secs = 196m, frequency = 915 MHz, antenna heights = 1.5m, large scale model uses two-ray ground reflection and its power calculation represents the mean squared value of the Ricean envelope.)

Since k is fixed, the time-spacing for a maximum Doppler frequency of f'_m will be

$$\Delta t' = \frac{1}{F'} = \frac{k}{f'_m} = \frac{f_{m_0}}{F_0 f'_m}. \tag{9}$$

Thus, given a data set created with a particular set of parameters, an arbitrary value of maximum Doppler frequency can be simulated by appropriately stretching or squeezing the original time series data set:

$$\Delta t' = \frac{f_{m_0}}{f'_m} \Delta t_0, \tag{10}$$

where Δt_0 and f_{m_0} are the time-spacing and maximum Doppler frequency of the dataset respectively, f'_m is the maximum Doppler frequency for the desired scenario and $\Delta t'$ is the resultant time-spacing.

4. Verification of Ricean statistics

Fig 1 shows packet arrivals as modeled by this implementation within the *ns* simulator. A small time interval from a simulation run consisting of UDP traffic between a stationary and a moving node is presented. The received signal strength of the packets obeys the correct amplitude statistics and has the appropriate time correlation. Packets
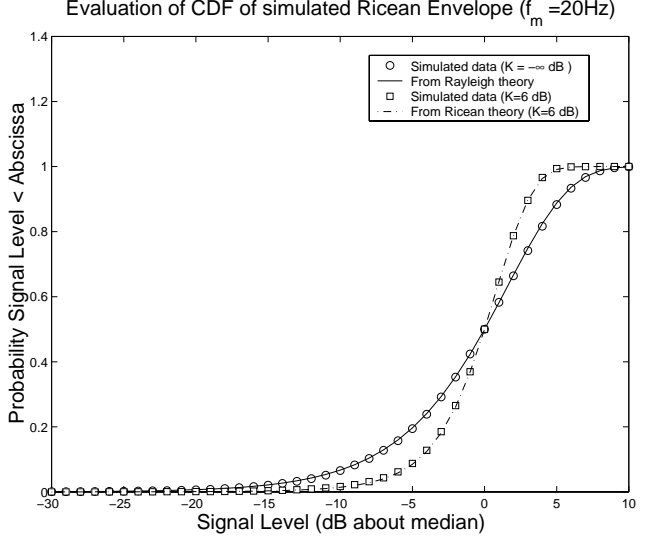
Figure 2. Cumulative distribution for simulated Ricean and Rayleigh Envelopes

can arrive at arbitrary times that may fall between dataset points. Interpolation has to be performed between dataset points before computing the envelope.

Some statistical metrics to verify the fading envelope are the level crossing rate (LCR), the cumulative distribution function (CDF), and the average fade duration (AFD).

The level crossing rate is the expected rate at which the envelope, normalized to the RMS signal level, crosses a specified level in a positive-going direction. The average fade duration is the average time for which the signal is below a specified level. The level crossing rate and average fade duration are important measures to ensure that a fading simulator fits the desired statistical behavior [6]. The level crossing rate for the Ricean envelope is
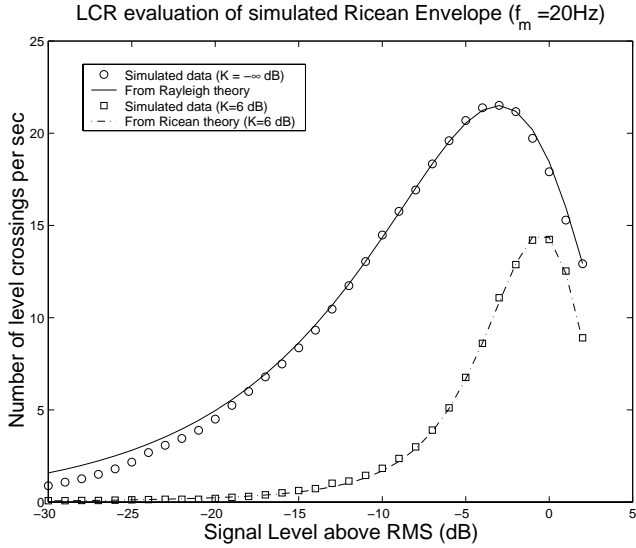
$$
\begin{aligned}
N_R &= \sqrt{\frac{\beta}{2\pi}} p(R) \\
&= \sqrt{\pi} f_m \frac{R}{\sigma} e^{-\left[\frac{R^2}{2\sigma^2} + \frac{A^2}{2\sigma^2}\right]} I_0\left(\frac{A}{\sigma}\frac{R}{\sigma}\right),
\end{aligned} \tag{11}
$$

where $\beta = 2\left(\pi\sigma f_m\right)^2$ and $p(r)$ is the probability density function for the Ricean envelope given by
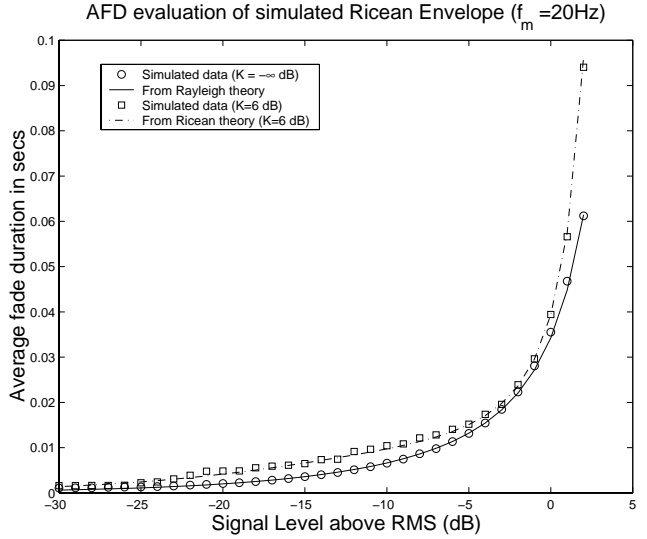
$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2 + A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right). \tag{12}$$

Using Eqn 2, the level crossing rate can finally be written in terms of the normalized signal envelope ρ:

$$
\begin{aligned}
N_R &= \sqrt{2\pi(K+1)} f_m \rho e^{-\left[\rho^2(K+1)+K\right]} \\
&\quad \times I_0\left(\sqrt{2K}\cdot\rho\sqrt{2(K+1)}\right),
\end{aligned} \tag{13}
$$

Figure 3. Level Crossing Rate for simulated Ricean and Rayleigh Envelopes

Figure 4. Average Fade Duration for simulated Ricean and Rayleigh Envelopes

where

$$\rho = \frac{R}{R_{rms}} = \frac{R}{\sigma\sqrt{2(K+1)}}. \tag{14}$$

The cumulative distribution function obtained by numerically integrating Eqn 12 is plotted in Fig 2 along with the CDF of the simulated Ricean envelope.

The average fade duration (average duration for which the signal level is below a specified level R) is given by

$$\bar{\tau} = \frac{Pr\,(r \leq R)}{N_R} \tag{15}$$

where $Pr\,(r \leq R)$ is the cumulative distribution function of the envelope.

The level crossing rate and the average fade duration for the simulated Ricean envelope are compared with the theoretical values in Fig 3 and in Fig 4 respectively.

5. Summary

This work demonstrates a simple and computationally efficient way to model small-scale fading statistics within a packet level simulator. Long simulation runs are accommodated by reusing the dataset. This approach allows the accurate modeling of all fades whose duration is small compared to the duration of the dataset.

References

[1] G. Arredondo, W. H. Chriss, and E. H. Walker. A multipath fading simulator for mobile radio. *IEEE Transactions on Vehicular Technology*, VT-22(4):241–244, November 1973.

[2] E. Casas and C. Leung. A Simple Digital Fading Simulator for Mobile Radio. *IEEE Transactions on Vehicular Technology*, 39(3):205–212, August 1990.

[3] R. H. Clarke. A Statistical Theory of Mobile-radio Reception. *Bell Systems Technical Journal*, 47:957–1000, 1968.

[4] R. A. Comroe. All-digital Rayleigh fading simulator. In *Proceedings of the National Electonics Conference*, volume 32, pages 136–139, 1978.

[5] M. J. Gans. A Power Spectral Theory of Propagation in the Mobile Radio Environment. *IEEE Transactions on Vehicular Technology*, VT-21:27–38, February 1972.

[6] M. Pätzold and F. Laue. Level-Crossing Rate and Average Duration of Fades of Deterministic Simulation Models for Rice Fading Channels. *IEEE Transactions on Vehicular Technology*, 48(4):1121–1129, July 1999.

[7] T. S. Rappaport. *Wireless Communications: Principles and Practice*, chapter 4, pages 181–185. Prentice Hall, 1996.

[8] J. I. Smith. A Computer Generated Multipath Fading Simulation for Mobile Radio. *IEEE Transactions on Vehicular Technology*, VT-24(3):39–40, August 1975.

[9] M. Tycz and M. W. Fitzmaurice. Channel simulation for optical communication systems. *Applied Optics*, 13(6):1283–1286, June 1974.